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DEGREE AND DISTANCE BASED TOPOLOGICAL INDICES OF SOME WHEEL RELATED GRAPHS

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Abstract. The degree distance index and Gutman index are the topological indices which are calculated by counting the degrees and distance between the vertices of a connected graph. In this paper, the degree distance and Gutman indices of some wheel related graphs such as helm graph, gear graph, friendship graph, flower graph, sunflower graph, and fan graph are calculated in terms of number of vertices of wheel graph.

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1. Introduction

The topological indices are the numerical parameters associated with a graph that are usually graph invariant. The topological index is also called as a molecular descriptor as it is a mathematical formula that can be applied to any graph which models some molecular structure. They can be classified by the properties of graphs such as degree, distance, eccentricity, status, non-incidence edges and so on. Among all the topological indices, Wiener index $W(G)$ of a graph G [12], defined as sum of the distance between all the pairs of vertices of a graph, is the first topological index to be used in chemistry to determine the boiling point of paraffin.

In this paper, the authors have considered two degree and distance based topological indices, namely degree distance index and Gutman index of a connected graph. The degree of a vertex u in a graph G is the number of edges incident on u and the notation is $\deg_G(u)$. The distance between two vertices u, v of a connected graph G is the number of edges in the shortest path between u and v. $d_G(u,v)$ denotes the distance between the vertices u and v in the graph G.

The degree distance index was considered by A. A. Dobrynin and A. A. Kochetova [3] and by I. Gutman [5] in 1994 with two different names. A. A. Dobrynin and A. A. Kochetova used the term degree distance index and I. Gutman used the term Schultz molecular topological index. The degree distance index of a connected graph G is defined as,

$$DD(G) = \sum_{1\leq i<j\leq n} (\deg_G(u_i) + \deg_G(u_j))\, d_G(u_i, u_j).$$

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In [5], I. Gutman introduced Schultz index of second type which is now termed as Gutman index. The Gutman index of a connected graph G is defined as,

$$Gut(G) = \sum_{1\le i<j\le n} \deg_G(u_i)\deg_G(u_j)d_G(u_i,u_j).$$

The relation between degree distance index and Gutman index was studied in [2, 4].

Few topological indices of wheel related graphs can be seen in [9, 10, 8]. Motivated by these studies, in this paper degree distance indices and Gutman indices of some wheel related graphs are studied.

2. Preliminaries

We recall some basic definitions in this section.

Definition 2.1. [6] *The wheel graph W_{n+1} is obtained by connecting a single universal vertex to all vertices of a cycle C_n.*

Definition 2.2. [11] *The helm graph H_{n+1} is obtained from wheel graph W_{n+1} by adding a pendant vertex to each n peripheral vertices.*

Definition 2.3. [1] *The gear graph G_{n+1} is obtained from W_{n+1} by inserting a vertex into each peripheral edge of W_{n+1}.*

Definition 2.4. [7] *The friendship graph F_{n+1} is obtained from removing $\frac{n}{2}$ alternate peripheral edges from wheel graph of odd order.*

Definition 2.5. [11] *The flower graph Fl_{n+1} is obtained from inserting a new vertex u_i' into each peripheral vertex u_i of the wheel graph and $u_i' \sim u_i$ and $u_i' \sim u_0$, where u_0 is the universal vertex of wheel graph.*

Definition 2.6. [11] *The sunflower graph SF_{n+1} is obtained from inserting a new vertex u_i' into each peripheral edge u_i of the wheel graph and u_i' is adjacent to the end vertices u_i.*

Definition 2.7. [11] *The fan graph $F_{n,1}$ is obtained from removing one peripheral edge from the wheel graph.*

Remark: It is easy to see that, the degree distance index, Gutman index, and Wiener index of a wheel graph W_{n+1} is $n(7n-9)$, $6n(2n-3)$, and $n(n-1)$, respectively.

3. Main results

In this section, the degree distance index and Gutman index of few wheel related graphs with number of vertices ≥ 4 are calculated in terms of degree distance index, Gutman index, and Wiener index of wheel graph and number of vertices of a wheel graph.

Theorem 3.1. *For a helm graph H_{n+1},*

(a) $DD(H_{n+1}) = 30n^2 - 36n$.
(b) $Gut(H_{n+1}) = 36n^2 - 51n$.

Proof. Let u_0 be the central vertex, $u_i,\ 1 \le i \le n$ be the peripheral vertices of the wheel graph, and u_i' be the pendent vertex adjacent to the vertex u_i, $1 \le i \le n$.

(a) Consider,

$$\begin{aligned} DD(u_0, u_i : H_{n+1}) &= \sum_{i=1}^{n} (\deg_{W_{n+1}}(u_0) + \deg_{W_{n+1}}(u_i) + 1) d_{W_{n+1}}(u_0, u_i) \\ &= DD(u_0, u_i : W_{n+1}) + n,\ 1 \le i \le n. \end{aligned}$$

$$\begin{aligned} DD(u_i, u_j : H_{n+1}) &= \sum_{1 \le i < j \le n} (\deg_{W_{n+1}}(u_i) + 1 + \deg_{W_{n+1}}(u_j) + 1) d_{W_{n+1}}(u_i, u_j) \\ &= DD(u_i, u_j : W_{n+1}) + 2\left(n + \left(\binom{n}{2} - n\right)(2)\right) \\ &= DD(u_i, u_j : W_{n+1}) + 2n^2 - 4n,\ 1 \le i < j \le n. \end{aligned}$$

$$DD(u_0, u_i' : H_{n+1}) = n(n+1)(2) = 2n^2 + 2n,\ 1 \le i \le n.$$

$$DD(u_i, u_i' : H_{n+1}) = n(4+1)(1) = 5n,\ 1 \le i \le n.$$

$$\begin{aligned} DD(u_i, u_j' : H_{n+1}) &= 2n(4+1)(2) + (n^2 - 3n)(4+1)(3) \\ &= 15n^2 - 25n,\ 1 \le i, j \le n, i \ne j. \end{aligned}$$

$$\begin{aligned} DD(u_i', u_j' : H_{n+1}) &= n(1+1)(3) + \left(\binom{n}{2} - n\right)(1+1)(4) \\ &= 4n^2 - 6n,\ 1 \le i < j \le n. \end{aligned}$$

Therefore,

$$\begin{aligned} DD(H_{n+1}) &= DD(W_{n+1}) + 23n^2 - 27n \\ &= 30n^2 - 36n. \end{aligned}$$

(b) Consider,

$$\begin{aligned} Gut(u_0, u_i : H_{n+1}) &= \sum_{i=1}^{n+1} (\deg_{W_{n+1}}(u_0))(\deg_{W_{n+1}}(u_i) + 1) d_{W_{n+1}}(u_0, u_i) \\ &= Gut(u_0, u_i : W_{n+1}) + n^2,\ 1 \le i \le n. \end{aligned}$$

$$\begin{aligned} Gut(u_i, u_j : H_{n+1}) &= \sum_{1 \le i < j \le n} (\deg_{W_{n+1}}(u_i) + 1)(\deg_{W_{n+1}}(u_j) + 1) d_{W_{n+1}}(u_i, u_j) \\ &= Gut(u_i, u_j : W_{n+1}) + (3+3+1)\left(n(1) + \binom{n}{2} - n\right)(2) \\ &= Gut(u_i, u_j : W_{n+1}) + 7n^2 - 14n,\ 1 \le i < j \le n. \end{aligned}$$

$$Gut(u_0, u_i' : H_{n+1}) = n(n+1)(2) = 2n^2 + 2n,\ 1 \le i \le n.$$

$$Gut(u_i, u_i' : H_{n+1}) = n(4)(1)(1) = 4n,\ 1 \le i \le n.$$

$$\begin{aligned} Gut(u_i, u'_j : H_{n+1}) &= 2n(4)(1)(2) + (n^2 - 3n)(4)(1)(3) \\ &= 12n^2 - 20n,\ 1 \leq i, j \leq n, i \neq j. \end{aligned}$$

$$\begin{aligned} Gut(u'_i, u'_j : H_{n+1}) &= n(1)(1)(3) + \left(\binom{n}{2} - n\right)(1)(1)(4) \\ &= 2n^2 - 3n,\ 1 \leq i < j \leq n. \end{aligned}$$

Therefore,

$$\begin{aligned} Gut(H_{n+1}) &= Gut(W_{n+1}) + 24n^2 - 33n \\ &= 36n^2 - 51n. \end{aligned}$$

□

Theorem 3.2. *For a gear graph G_{n+1},*
(a) $DD(G_{n+1}) = 32n^2 - 35n$.
(b) $Gut(G_{n+1}) = 42n^2 - 49n$.

Proof. Let u_0 be the central vertex and u_i, $1 \leq i \leq n$ be the peripheral vertices of the wheel graph. Let u'_i be the vertex inserted to the edge u_iu_{i+1}, $1 \leq i \leq n-1$ and u'_n be the vertex inserted to the edge u_1u_n.

(a) Consider,

$$\begin{aligned} DD(u_0, u_i : G_{n+1}) &= \sum_{i=1}^{n} (\deg_{W_{n+1}}(u_0) + \deg_{W_{n+1}}(u_i)) d_{W_{n+1}}(u_0, u_i) \\ &= DD(u_0, u_i : W_{n+1}),\ 1 \leq i \leq n. \end{aligned}$$

$$\begin{aligned} DD(u_i, u_j : G_{n+1}) &= \sum_{i=1}^{n-1} (\deg_{W_{n+1}}(u_i) + \deg_{W_{n+1}}(u_{i+1}))(d_{W_{n+1}}(u_i, u_{i+1}) + 1) \\ &\quad + (\deg_{W_{n+1}}(u_1) + \deg_{W_{n+1}}(u_n))(d_{W_{n+1}}(u_1, u_n) + 1) \\ &\quad + \sum_{\substack{1 \leq i < j \leq n \\ u_iu_j \notin E(W_{n+1})}} (\deg_{W_{n+1}}(u_i) + \deg_{W_{n+1}}(u_j))(d_{W_{n+1}}(u_i, u_j)) \\ &= DD(u_i, u_j : W_{n+1}) + 6n,\ 1 \leq i < j \leq n. \end{aligned}$$

$$DD(u_0, u'_i : G_{n+1}) = n(n+2)(2) = 2n^2 + 4n,\ 1 \leq i \leq n.$$

$$\begin{aligned} DD(u_i, u'_j : G_{n+1}) &= 2n(3+2)(1) + (n^2 - 2n)(3+2)(3) \\ &= 15n^2 - 20n,\ 1 \leq i, j \leq n. \end{aligned}$$

$$\begin{aligned} DD(u'_i, u'_j : G_{n+1}) &= n(2+2)(2) + \left(\binom{n}{2} - n\right)(2+2)(4) \\ &= 8n^2 - 16n,\ 1 \leq i < j \leq n. \end{aligned}$$

Therefore,

$$\begin{aligned} DD(G_{n+1}) &= DD(W_{n+1}) + 25n^2 - 26n \\ &= 32n^2 - 35n. \end{aligned}$$

(b) Consider,

$$Gut(u_0, u_i : G_{n+1}) = \sum_{i=1}^{n}(\deg_{W_{n+1}}(u_0))(\deg_{W_{n+1}}(u_i))d_{W_{n+1}}(u_0, u_i)$$
$$= Gut(u_0, u_i : W_{n+1}),\ 1 \leq i \leq n.$$

$$Gut(u_i, u_j : G_{n+1}) = \sum_{i=1}^{n-1}(\deg_{W_{n+1}}(u_i))(\deg_{W_{n+1}}(u_{i+1}))(d_{W_{n+1}}(u_i, u_{i+1}) + 1)$$
$$+ (\deg_{W_{n+1}}(u_1))(\deg_{W_{n+1}}(u_n))(d_{W_{n+1}}(u_1, u_n) + 1)$$
$$+ \sum_{\substack{1\leq i<j\leq n \\ u_iu_j \notin E(W_{n+1})}} (\deg_{W_{n+1}}(u_i))(\deg_{W_{n+1}}(u_j))(d_{W_{n+1}}(u_i, u_j))$$
$$= Gut(u_i, u_j : W_{n+1}) + 9n,\ 1 \leq i < j \leq n.$$

$$Gut(u_0, u'_i : G_{n+1}) = n(n)(2)(2) = 4n^2,\ 1 \leq i \leq n.$$

$$Gut(u_i, u'_j : G_{n+1}) = 2n(3)(2)(1) + (n^2 - 2n)(3)(2)(3)$$
$$= 18n^2 - 24n,\ 1 \leq i, j \leq n.$$

$$Gut(u'_i, u'_j : G_{n+1}) = n(2+2)(2) + \left(\binom{n}{2} - n\right)(2+2)(4)$$
$$= 8n^2 - 16n,\ 1 \leq i < j \leq n.$$

Therefore,

$$Gut(G_{n+1}) = Gut(W_{n+1}) + 30n^2 - 31n$$
$$= 42n^2 - 49n.$$

□

Theorem 3.3. *For a friendship graph F_{n+1},*

(a) $DD(F_{n+1}) = 5n^2 - 4n.$
(b) $Gut(F_{n+1}) = 6n^2 - 6n.$

Proof. Let u_0 be the central vertex and $u_i,\ 1 \leq i \leq n$ be the peripheral vertices of the friendship graph.

(a) Consider,

$$DD(u_0, u_i : F_{n+1}) = \sum_{i=1}^{n}(\deg_{W_{n+1}}(u_0) + \deg_{W_{n+1}}(u_i) - 1)d_{W_{n+1}}(u_0, u_i)$$
$$= DD(u_0, u_i : W_{n+1}) - n,\ 1 \leq i \leq n.$$

$$\begin{aligned}DD(u_i,u_j:F_{n+1}) &= \sum_{\substack{u_iu_j\in E(W_{n+1})\\ u_iu_j\in E(F_{n+1})}} (\deg_{W_{n+1}}(u_i)-1+\deg_{W_{n+1}}(u_j)-1)(d_{W_{n+1}}(u_i,u_j))\\ &+ \sum_{\substack{u_iu_j\in E(W_{n+1})\\ u_iu_j\notin E(F_{n+1})}} (\deg_{W_{n+1}}(u_i)-1+\deg_{W_{n+1}}(u_j)-1)(d_{W_{n+1}}(u_i,u_j)+1)\\ &+ \sum_{\substack{u_iu_j\notin E(W_{n+1})\\ u_iu_j\notin E(F_{n+1})}} (\deg_{W_{n+1}}(u_i)-1+\deg_{W_{n+1}}(u_j)-1)(d_{W_{n+1}}(u_i,u_j))\\ &= DD(u_i,u_j:W_{n+1})-n+DD(u_i,u_j:W_{n+1})+n+DD(u_i,u_j:W_{n+1})\\ &-2n^2+6n\\ &= DD(u_i,u_j:W_{n+1})-2n^2+6n,\ 1\le i<j\le n.\end{aligned}$$

Therefore,

$$\begin{aligned}DD(F_{n+1}) &= DD(W_{n+1})-2n^2+5n\\ &= 5n^2-4n.\end{aligned}$$

(b) Consider,

$$\begin{aligned}Gut(u_0,u_i:F_{n+1}) &= \sum_{i=1}^{n}(\deg_{W_{n+1}}(u_0))(\deg_{W_{n+1}}(u_i)-1)d_{W_{n+1}}(u_0,u_i)\\ &= Gut(u_0,u_i:W_{n+1})-n^2,\ 1\le i\le n.\end{aligned}$$

$$\begin{aligned}Gut(u_i,u_j:F_{n+1}) &= \sum_{\substack{u_iu_j\in E(W_{n+1})\\ u_iu_j\in E(F_{n+1})}} (\deg_{W_{n+1}}(u_i)-1)(\deg_{W_{n+1}}(u_j)-1)(d_{W_{n+1}}(u_i,u_j))\\ &+ \sum_{\substack{u_iu_j\in E(W_{n+1})\\ u_iu_j\notin E(F_{n+1})}} (\deg_{W_{n+1}}(u_i)-1)(\deg_{W_{n+1}}(u_j)-1)(d_{W_{n+1}}(u_i,u_j)+1)\\ &+ \sum_{\substack{u_iu_j\notin E(W_{n+1})\\ u_iu_j\notin E(F_{n+1})}} (\deg_{W_{n+1}}(u_i)-1)(\deg_{W_{n+1}}(u_j)-1)(d_{W_{n+1}}(u_i,u_j))\\ &= Gut(u_i,u_j:W_{n+1})-DD(W_{n+1})-n(n+3)+W(W_{n+1})-n\\ &+\frac{9n}{2}-\frac{3n}{2}-\frac{3n}{2}+\frac{n}{2}\\ &= Gut(u_i,u_j:W_{n+1})-DD(W_{n+1})+W(W_{n+1})+4n,\ 1\le i<j\le n.\end{aligned}$$

Therefore,

$$\begin{aligned}Gut(F_{n+1}) &= Gut(W_{n+1})-DD(W_{n+1})+W(W_{n+1})+4n\\ &= 6n^2-6n.\end{aligned}$$

□

Theorem 3.4. *For a flower graph Fl_{n+1},*

(a) $DD(Fl_{n+1}) = 28n^2-20n.$
(b) $Gut(Fl_{n+1}) = 48n^2-44n.$

Proof. Let u_0 be the central vertex, $u_i,\ 1\le i\le n$ be the peripheral vertices of the wheel graph, and u_i' be the pendent vertex attached to the vertex u_i, $1\le i\le n$.

(a) Consider,

$$DD(u_0, u_i : Fl_{n+1}) = \sum_{i=1}^{n}(2\deg_{W_{n+1}}(u_0) + \deg_{W_{n+1}}(u_i) + 1)d_{W_{n+1}}(u_0, u_i)$$
$$= DD(u_0, u_i : W_{n+1}) + n^2 + n, \ 1 \le i \le n.$$

$$DD(u_i, u_j : Fl_{n+1}) = \sum_{1\le i<j\le n}(\deg_{W_{n+1}}(u_i) + 1 + \deg_{W_{n+1}}(u_j) + 1)d_{W_{n+1}}(u_i, u_j)$$
$$= DD(u_i, u_j : W_{n+1}) + 2\left(n(1) + \left(\binom{n}{2} - n\right)(2)\right)$$
$$= DD(u_i, u_j : W_{n+1}) + 2n^2 - 4n, \ 1 \le i < j \le n.$$

$$DD(u_0, u_i' : Fl_{n+1}) = n(2n+2)(1) = 2n^2 + 2n, \ 1 \le i \le n.$$

$$DD(u_i, u_j' : Fl_{n+1}) = n(4+2)(1) + (n^2 - n)(4+2)(2)$$
$$= 12n^2 - 6n, \ 1 \le i, j \le n.$$

$$DD(u_i', u_j' : Fl_{n+1}) = \binom{n}{2}(2+2)(2) = 4n^2 - 4n, \ 1 \le i < j \le n.$$

Therefore,

$$DD(Fl_{n+1}) = DD(W_{n+1}) + 21n^2 - 11n$$
$$= 28n^2 - 20n.$$

(b) Consider,

$$Gut(u_0, u_i : Fl_{n+1}) = \sum_{i=1}^{n}(2\deg_{W_{n+1}}(u_0))(\deg_{W_{n+1}}(u_i) + 1)d_{W_{n+1}}(u_0, u_i)$$
$$= Gut(u_0, u_i : W_{n+1}) + 5n^2, \ 1 \le i \le n.$$

For $1 \le i < j \le n$,

$$Gut(u_i, u_j : Fl_{n+1}) = \sum_{1\le i<j\le n}(\deg_{W_{n+1}}(\deg_{W_{n+1}}(u_i) + 1)(\deg_{W_{n+1}}(u_j) + 1)d_{W_{n+1}}(u_i, u_j)$$
$$= DD(u_i, u_j : W_{n+1}) + DD(W_{n+1}) - n(n+3) + W(W_{n+1}) - n.$$

$$Gut(u_0, u_i' : Fl_{n+1}) = n(2n)(2)(1) = 4n^2, \ 1 \le i \le n.$$

$$Gut(u_i, u_j' : Fl_{n+1}) = n(4)(2)(1) + (n^2 - n)(4)(2)(2)$$
$$= 16n^2 - 8n, \ 1 \le i, j \le n.$$

$$Gut(u_i', u_j' : Fl_{n+1}) = \binom{n}{2}(2)(2)(2) = 4n^2 - 4n, \ 1 \le i < j \le n.$$

Therefore,

$$Gut(Fl_{n+1}) = Gut(W_{n+1}) + DD(W_{n+1}) + W(W_{n+1}) + 28n^2 - 16n$$
$$= 48n^2 - 44n.$$

□

Theorem 3.5. *For a sunflower graph SF_{n+1},*

(a) $DD(SF_{n+1}) = 42n^2 - 73n.$

(b) $Gut(SF_{n+1}) = 72n^2 - 130n$.

Proof. Let u_0 be the central vertex, $u_i,\ 1 \le i \le n$ be the peripheral vertices of the wheel graph, and u'_i be the pendent vertex attached to the vertex u_i, $1 \le i \le n$.

(a) Consider,

$$\begin{aligned} DD(u_0, u_i : SF_{n+1}) &= \sum_{i=1}^{n} (\deg_{W_{n+1}}(u_0) + \deg_{W_{n+1}}(u_i) + 2) d_{W_{n+1}}(u_0, u_i) \\ &= DD(u_0, u_i : W_{n+1}) + 2n,\ 1 \le i \le n. \end{aligned}$$

$$\begin{aligned} DD(u_i, u_j : SF_{n+1}) &= \sum_{1 \le i < j \le n} (\deg_{W_{n+1}}(u_i) + 2 + \deg_{W_{n+1}}(u_j) + 2) d_{W_{n+1}}(u_i, u_j) \\ &= DD(u_i, u_j : W_{n+1}) + 4\left(n + \left(\binom{n}{2} - n\right)(2)\right) \\ &= DD(u_i, u_j : W_{n+1}) + 2n^2 - 8n,\ 1 \le i < j \le n. \end{aligned}$$

$$DD(u_0, u'_i : SF_{n+1}) = n(n+2)(2) = 2n^2 + 4n,\ 1 \le i \le n.$$

$$\begin{aligned} DD(u_i, u'_j : SF_{n+1}) &= 2n(5+2)(1) + 2n(5+2)(2) + (n^2 - 4n)(5+2)(3) \\ &= 21n^2 - 42n,\ 1 \le i, j \le n. \end{aligned}$$

$$\begin{aligned} DD(u'_i, u'_j : SF_{n+1}) &= n(2+2)(2) + n(2+2)(3) + \left(\binom{n}{2} - 2n\right)(2+2)(4) \\ &= 8n^2 - 20n,\ 1 \le i < j \le n. \end{aligned}$$

Therefore,

$$\begin{aligned} DD(SF_{n+1}) &= DD(W_{n+1}) + 35n^2 - 64n \\ &= 42n^2 - 73n. \end{aligned}$$

(b) Consider,

$$\begin{aligned} Gut(u_0, u_i : SF_{n+1}) &= \sum_{i=1}^{n} (\deg_{W_{n+1}}(u_0))(\deg_{W_{n+1}}(u_i) + 2) d_{W_{n+1}}(u_0, u_i) \\ &= Gut(u_0, u_i : W_{n+1}) + 2n^2,\ 1 \le i \le n. \end{aligned}$$

For $1 \le i < j \le n$,

$$\begin{aligned} Gut(u_i, u_j : SF_{n+1}) &= \sum_{1 \le i < j \le n} (\deg_{W_{n+1}}(u_i) + 2)(\deg_{W_{n+1}}(u_j) + 2) d_{W_{n+1}}(u_i, u_j) \\ &= Gut(u_i, u_j : W_{n+1}) + 2DD(W_{n+1}) - 2n(n+3) + 4W(W_{n+1}) - 4n. \end{aligned}$$

$$Gut(u_0, u'_i : SF_{n+1}) = n(n)(2)(2) = 4n^2,\ 1 \le i \le n.$$

$$\begin{aligned} Gut(u_i, u'_j : SF_{n+1}) &= 2n(5)(2)(1) + 2n(5)(2)(2) + (n^2 - 4n)(5)(2)(3) \\ &= 30n^2 - 60n,\ 1 \le i, j \le n. \end{aligned}$$

$$\begin{aligned} Gut(u'_i, u'_j : SF_{n+1}) &= n(2)(2)(2) + n(2)(2)(3) + \left(\binom{n}{2} - 2n\right)(2)(2)(4) \\ &= 8n^2 - 20n,\ 1 \le i < j \le n. \end{aligned}$$

Therefore,

$$\begin{aligned} Gut(SF_{n+1}) &= Gut(W_{n+1}) + 2DD(W_{n+1}) + 4W(W_{n+1}) + 42n^2 - 90n \\ &= 72n^2 - 130n. \end{aligned}$$

□

Theorem 3.6. *For a fan graph* $F_{n,1}$,
(a) $DD(F_{n,1}) = 7n^2 - 13n + 10$.
(b) $Gut(F_{n,1}) = 12n^2 - 32n + 29$.

Proof. Let u_0 be the central vertex and $u_i,\ 1 \leq i \leq n$ be the peripheral vertices of the wheel graph. Let u_1u_n be the removed edge from wheel graph to obtain fan graph.

(a) Consider,

$$\begin{aligned} DD(u_0, u_i : F_{n,1}) &= \sum_{i=2}^{n-1} (\deg_{W_{n+1}}(u_0) + \deg_{W_{n+1}}(u_i)) d_{W_{n+1}}(u_0, u_i) \\ &+ (\deg_{W_{n+1}}(u_0) + \deg_{W_{n+1}}(u_1) - 1) d_{W_{n+1}}(u_0, u_1) \\ &+ (\deg_{W_{n+1}}(u_0) + \deg_{W_{n+1}}(u_n) - 1) d_{W_{n+1}}(u_0, u_n) \\ &= DD(u_0, u_i : W_{n+1}) - 2,\ 1 \leq i \leq n. \end{aligned}$$

$$\begin{aligned} DD(u_i, u_j : F_{n,1}) &= \sum_{2 \leq i < j \leq n-1} (\deg_{W_{n+1}}(u_i) + \deg_{W_{n+1}}(u_j)) d_{W_{n+1}}(u_i, u_j) \\ &+ (\deg_{W_{n+1}}(u_1) - 1 + \deg_{W_{n+1}}(u_n) - 1)(d_{W_{n+1}}(u_1, u_n) + 1) \\ &+ (\deg_{W_{n+1}}(u_1) - 1 + \deg_{W_{n+1}}(u_2)) d_{W_{n+1}}(u_1, u_2) \\ &+ (\deg_{W_{n+1}}(u_{n-1}) + \deg_{W_{n+1}}(u_n) - 1) d_{W_{n+1}}(u_{n-1}, u_n) \\ &= DD(u_i, u_j : W_{n+1}) - 4n + 12,\ 1 \leq i < j \leq n. \end{aligned}$$

Therefore,

$$\begin{aligned} DD(F_{n,1}) &= DD(W_{n+1}) - 4n + 10 \\ &= 7n^2 - 13n + 10. \end{aligned}$$

(b) Consider,

$$\begin{aligned} Gut(u_0, u_i : F_{n,1}) &= \sum_{i=2}^{n} (\deg_{W_{n+1}}(u_0))(\deg_{W_{n+1}}(u_i)) d_{W_{n+1}}(u_0, u_i) \\ &+ (\deg_{W_{n+1}}(u_0))(\deg_{W_{n+1}}(u_1) - 1) d_{W_{n+1}}(u_0, u_1) \\ &+ (\deg_{W_{n+1}}(u_0))(\deg_{W_{n+1}}(u_n) - 1) d_{W_{n+1}}(u_0, u_n) \\ &= Gut(u_0, u_i : W_{n+1}) - 2n,\ 1 \leq i \leq n. \end{aligned}$$

$$\begin{aligned} Gut(u_i, u_j : F_{n,1}) &= \sum_{2 \leq i < j \leq n-1} (\deg_{W_{n+1}}(u_i))(\deg_{W_{n+1}}(u_j)) d_{W_{n+1}}(u_i, u_j) \\ &+ (\deg_{W_{n+1}}(u_1) - 1)(\deg_{W_{n+1}}(u_n) - 1)(d_{W_{n+1}}(u_i, u_j) + 1) \\ &+ (\deg_{W_{n+1}}(u_1) - 1)(\deg_{W_{n+1}}(u_2)) d_{W_{n+1}}(u_1, u_2) \\ &+ (\deg_{W_{n+1}}(u_{n-1}))(\deg_{W_{n+1}}(u_n) - 1) d_{W_{n+1}}(u_{n-1}, u_n) \\ &= DD(u_i, u_j : W_{n+1}) - 12n + 29,\ 1 \leq i < j \leq n. \end{aligned}$$

Therefore,

$$\begin{aligned} Gut(F_{n,1}) &= Gut(W_{n+1}) - 14n + 29 \\ &= 12n^2 - 32n + 29. \end{aligned}$$

□

Remark: It is observed that,

(a) $DD(SF_{n+1}) > DD(G_{n+1}) > DD(Fl_{n+1}) > DD(H_{n+1}) > DD(F_{n,1}) > DD(F_{n+1})$, for $4 \leq n \leq 7$.
(b) $DD(SF_{n+1}) > DD(G_{n+1}) > DD(Fl_{n+1}) = DD(H_{n+1}) > DD(F_{n,1}) > DD(F_{n+1})$, for $n = 8$.
(c) $DD(SF_{n+1}) > DD(G_{n+1}) > DD(H_{n+1}) > DD(Fl_{n+1}) > DD(F_{n,1}) > DD(F_{n+1})$, for $n \geq 9$.
(d) $Gut(SF_{n+1}) > Gut(Fl_{n+1}) > Gut(G_{n+1}) > Gut(H_{n+1}) > Gut(F_{n,1}) > Gut(F_{n+1})$, for $n \geq 4$.

4. Conclusion

In this paper, the authors have obtained degree distance index and Gutman index of some wheel related graphs such as helm graph, gear graph, friendship graph, flower graph, sunflower graph, and fan graph. The degree distance and Gutman indices of these wheel related graphs are in terms of number of vertices of wheel graph.

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